Expanding brackets

An investigation…

Expand $(x + y)^0$

Easy!

Anything "to the power 0" is 1.

Answer:
1

Expand $(x + y)^1$

Easy!

That's just itself – it doesn't change.

Answer:

$x + y$

Expand $(x + y)^2$

Still easy!

$\times$	x	$+y$
x	x^2	$+xy$
$+y$	$+xy$	y^2

Answer:

$x^2 + 2xy + y^2$

Expand $(x + y)^3$

Not so easy!

Where did this line come from?

$\times$	x^2	$+2xy$	y^2
x	$\boldsymbol{x^3}$	$\boldsymbol{+2x^2y}$	$\boldsymbol{+xy^2}$
$+y$	$\boldsymbol{+x^2y}$	$\boldsymbol{+2xy^2}$	$\boldsymbol{y^3}$

Answer:

$\boldsymbol{x^3 + 3x^2y + 3xy^2 + y^3}$

Expand $(x + y)^4$

There must be an easier way!!!!

Are there any patterns in our results?

Let's have a look…

Our previous results:

$(x+y)^0$ $\quad 1$

$(x+y)^1$ $\quad x + y$

$(x+y)^2$ $\quad x^2 + 2xy + y^2$

$(x+y)^3$ $\quad x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4$ $\quad \mathbf{x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4}$

Can you predict what the next row will look like?

Can you explain how you got your result?

The answer to both those questions is "Yes" by the way, or at the very least you can have a go!

The coefficients make Pascal's Triangle:

"To the power 0"
"To the power 1"
"To the power 2"
"To the power 3"
"To the power 4"

Can you spot any patterns?

How do you get the next row?

				1						
			1		1					
		1		2		1				
	1		3		3		1			
1		4		6		4		1		
	1		5		10		10		5		1
1		6		15		20		15		6		1
	1		7		21		35		35		21		7		1
1		8		28		56		70		56		28		8		1
	1		9		36		84		126		126		84		36		9		1
1		10		45		120		210		252		210		120		45		10		1

What is a coefficient you say?

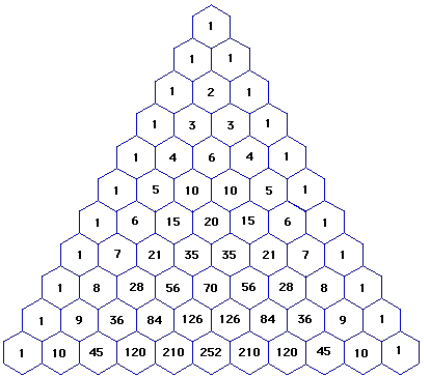
For example: $6x^5$

The coefficient

Blaise Pascal
1623-1662

Pascal was a “thief”!

Whilst this triangle bears Pascal’s name (from the 17th century) it is clear that both Chinese and Persian mathematicians were using it as early as the 11th century.



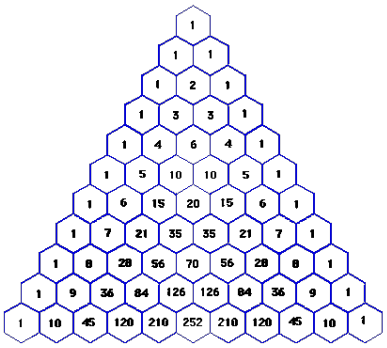
Using Pascal's Triangle…

Expand $(x + y)^5$

Answer:

$$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

Pascal's Triangle:



Try this…

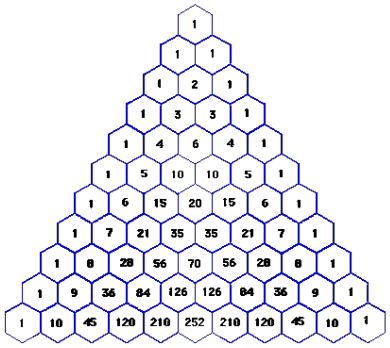
Expand $(x + y)^7$

Answer:

$$x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

What can you say about the powers of each letter?

Pascal's Triangle:



Can you use your previous result to expand $(\mathbf{1}+\mathbf{2}\boldsymbol{x})^{\mathbf{5}}$?

Expand $(\mathbf{1}+\mathbf{2}\boldsymbol{x})^{\mathbf{5}}$

Answer:

$1^5 + 5 \times 1^4 \times (2x) + 10 \times 1^3 \times (2x)^2 + 10 \times 1^2 \times (2x)^3 + 5 \times 1 \times (2x)^4 + (2x)^5$

Simplified:

$\mathbf{1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5}$

Previous answer:

Expansion of $(x+y)^5$:

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

How about expanding $(\mathbf{1-2x})^{\mathbf{5}}$?

Expand $(\mathbf{1-2x})^{\mathbf{5}}$

Answer:

$1^5 - 5 \times 1^4 \times (2x) + 10 \times 1^3 \times (2x)^2 - 10 \times 1^2 \times (2x)^3 + 5 \times 1 \times (2x)^4 - (2x)^5$

Simplified:

$\mathbf{1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5}$

Previous answer:

Expansion of $(1+2x)^5$:

$1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$

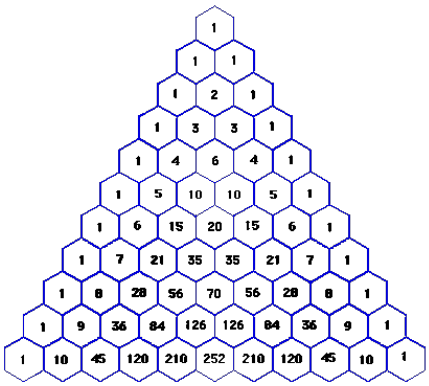
Can you explain why this happens?

Now try expanding these:

1. $(1-3x)^3$

2. $(2+5x)^4$

3. $(2x-3y)^6$



Answers:

1. $1 - 9x + 27x^2 - 27x^3$

2. $16 + 160x + 600x^2 + 1000x^3 + 625x^4$

3. $64x^6 - 576x^5y + 2160x^4y^2 - 4320x^3y^3 + 4860x^2y^4 - 2916xy^5 + 729y^6$

Investigation

This is a branch of mathematics that is concerned by the binomial theorem. It has useful applications in probability, angle identities, weather forecasting, economics and abstract algebra.

Imagine I take out a mortgage that takes n years to pay back. I will pay a rate of interest of this amount that is compounded annually, let us call that *i.*

Therefore, I can model the interest payments as $\left(1+\frac{100+i}{100}\right)^{n}$.

I can use this to help me work out the size of payments for any interest rate, over any number of years. Therefore, I could work out that if I wanted to buy a house worth £400000, paid over 30 years, what level of interest I would have to pay? This is what comparison websites do for different banks, and last time I checked, those businesses were very valuable!!!

What would the above interest payment formula look like for 4 years?

Incredibly tricky…(use software to help)…what would the above interest payment formula look like for 30 years!